Double Pipe Heat Exchanger Efficiency

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ABSTRACT

The purpose of this paper is to derive mathematical relationships which describe the efficiency for both parallel and counter flow double pipe heat exchangers. The efficiency is found to be dependent on the magnitude of the heat capacity ratio, and the number of transfer units, for both parallel flow and counter flow heat exchangers. The counter flow double pipe heat exchanger efficiency is found to be increases by increasing of the heat capacity ratio or by decreasing of the number of transfer units. The parallel flow double pipe heat exchanger efficiency is found to be increases by decreasing of both heat capacity ratio and number of transfer units. The counter flow double pipe heat exchangers efficiency is greater than that for the parallel flow double pipe heat exchanger at different values of heat capacity ratio and number of transfer units. An experimental work was performed on a double pipe heat exchanger to validate an analytical solution results. Results show an agreement between analytical solution and experimental work.

Keywords: Heat exchanger, Efficiency, Double pipe

1. Introduction

A typical double pipe heat exchanger consists of one pipe placed concentrically inside another of a larger diameter pipe with appropriate fittings to direct the flow from one section to the next. The major use of double pipe heat exchanger is for sensible heating or cooling of process fluids where small heat transfer area (up to $50m^2$) are required [1].

The concept of efficiency is used in many areas, particularly engineering, to assess the performance of real components and systems. Efficiency is a comparison between the actual (real) and ideal (best) performances and is typically defined to be less than or at best equal to 1. The ideal behaviour is generally known from modelling, and the limitations dictated by physical laws, particularly the second law of thermodynamics. Knowing the ideal performance, the actual performance can be determined if expressions for the efficiency as a function of the system characteristics and the operating conditions are known. The concept of heat exchanger efficiency provides a new way for the design and analysis of heat exchangers and heat exchanger networks [2].

The present study was undertaken to derive governing mathematical relationships, which describe the efficiency for both parallel and counter flow double pipe heat exchangers depending on the heat capacity ratio, and the number of transfer units. The experimental work was performed on a double pipe heat exchanger to validate an analytical solution results.

2. Governing Equations

The heat exchanger effectiveness ε is defined as the ratio of the actual heat transfer rate for a heat exchanger q to the maximum possible heat transfer rate q_{max} [3]:

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_h(T_{h,i}-T_{h,o})}{C_{min}(T_{h,i}-T_{c,i})} = \frac{C_c(T_{c,o}-T_{c,i})}{C_{min}(T_{h,i}-T_{c,i})} \qquad (1)$$

Where T temperature, h hot fluid, c cold fluid, o outlet, i inlet, min minimum, and C heat capacity rate. From equation (1)

$$T_{h,o} = T_{h,i} - \frac{\varepsilon C_{min}(T_{h,i}-T_{c,i})}{C_h} \qquad (2)$$

$$T_{c,o} = T_{c,i} + \frac{\varepsilon C_{min}(T_{h,i}-T_{c,i})}{C_c} \qquad (3)$$

The heat exchanger efficiency η is defined as the ratio of the actual rate of heat transfer in the heat exchanger q to the optimum rate of heat transfer q_o [2]:

$$\eta = \frac{q}{q_o} = \frac{q}{UA(\overline{T_h}-\overline{T_c})} \qquad (4)$$

Where U overall heat transfer coefficient, A heat transfer area, $\overline{T_h}$ and $\overline{T_c}$ average temperature of hot and cold fluid.

$$\overline{T_h} - \overline{T_c} = \frac{(T_{h,i}+T_{h,o})}{2} - \frac{(T_{c,i}+T_{c,o})}{2} \qquad (5)$$

$T_{h,o}$ and $T_{c,o}$ from equations (2) and (3) can be substituted into equation (5) to find another expression for $\overline{T_h} - \overline{T_c}$:

$$\overline{T_h} - \overline{T_c} = \left(T_{h,i} - T_{c,i}\right)\left[1 - \frac{1}{2}\,\varepsilon(C_r + 1)\right] \qquad (6)$$

Where C_r heat capacity ratio (= C_{min} / C_{max}), and max maximum. After solving equations (1) and (4), the general expression for η is:

$$\eta = \frac{\varepsilon}{NTU\left[1-\frac{1}{2}\varepsilon(C_r+1)\right]} \qquad (7)$$

Where NTU number of heat transfer units (= UA / C_{min}). From equation (7), the parallel flow double pipe heat exchanger efficiency is:

$$\eta = \frac{2(1-e^{-\alpha})}{NTU(C_r+1)(1+e^{-\alpha})} \qquad (8)$$

Where α variable $(=\mathrm{NTU}(\mathrm{C_r}+1))$. From equation (7), the counter flow double pipe heat exchanger efficiency for $C_r<1$ is:

$$\eta=\frac{2(1-e^{-\gamma})}{\mathrm{NTU}[2(1-C_r\,e^{-\gamma})-(1-e^{-\gamma})(C_r+1)]} \quad (9)$$

Where γ variable $(=\mathrm{NTU}(1-\mathrm{C_r}))$. For $C_r=1$, the counter flow double pipe heat exchanger efficiency is:

$$\eta=\frac{2}{2\mathrm{NTU}+2-\mathrm{NTU}(C_r+1)} \quad (10)$$

To obtain the Reynolds numbers for both inner tube and annulus of the double pipe heat exchanger Re_i and Re_o respectively, the following relations presented in [3] were used:

$$\mathrm{Re_i}=\frac{4m_h}{\pi\mu d_i} \quad (11)$$

$$\mathrm{Re_o}=\frac{4m_c}{\pi\mu(D_i+d_o)} \quad (12)$$

Where m_h and m_c mass flow rate of hot and cold water (m= ρQ), ρ density, Q volume flow rate, µ viscosity of water, d_i and d_o inner and outer diameter of inner tube, and D_i annulus inner diameter. The Nusselt numbers Nu are calculated from correlations presented in [1]. For laminar flow in circular annular duct:

$$\mathrm{Nu}=\mathrm{Nu}_\infty+\left[1+0.14\left(\frac{d_o}{D_i}\right)^{-0.5}\right]\frac{0.19\left(\frac{Pe_bD_h}{L}\right)^{0.8}}{1+0.117\left(\frac{Pe_bD_h}{L}\right)^{0.467}} \quad (13)$$

Where Pe Peclet number (=Re Pr), Pr Prandtl number, D_h hydraulic diameter of a tube, L length of a circular annulus, b bulk mean temperature, and Nu_∞ Nusselt number for fully developed flow, and the outer wall of the annulus is insulated which is given by:

$$\mathrm{Nu}_\infty=3.66+1.2\left(\frac{d_o}{D_i}\right)^{-0,5}$$

$$D_h=D_i-d_o$$

For turbulent flow:

$$\mathrm{Nu}=\frac{(f/2)Re_bPr_b}{1.07+12.7(f/2)^{0.5}(Pr_b{}^{2/3}-1)} \quad (14)$$

Equation (14) predicts the results in the range $10^4<Re_b<5\times10^6$ and $0.5<Pr_b<2000$. In the transition region where the Reynolds numbers are between 2300 and 10^4:

$$\mathrm{Nu} = \frac{(f/2)(\mathrm{Re_b} - 1000)\mathrm{Pr_b}}{1 + 12.7(f/2)^{0.5}(\mathrm{Pr_b}^{2/3} - 1)} \qquad (15)$$

$$f = (1.58 \ln \mathrm{Re_b} - 3.28)^{-2}$$

Where f friction factor. The hot and cold water outlet temperatures were calculated from the following mathematical relationships presented in [4]:

$$\theta_h = \frac{\exp(\gamma) - C_r \exp(\gamma X)}{\exp(\gamma) - C_r} \qquad (16)$$

$$\theta_c = \frac{\exp(\gamma) - \exp(\gamma X)}{\exp(\gamma) - C_r} \qquad (17)$$

$$\theta_h = \frac{1}{1 + C_r}[1 + C_r \exp(-\alpha X)] \qquad (18)$$

$$\theta_c = \frac{1}{1 + C_r}[1 - \exp(-\alpha X)] \qquad (19)$$

Where θ_h and θ_c dimensionless temperature of hot and cold fluid, and X dimensionless distance in x direction (= x / L), x = L at the outlet of hot fluid.

$$\theta = \frac{T - T_{c,i}}{T_{h,i} - T_{c,i}}$$

Equations (16), (17), (18), and (19) using when the heat capacity rate of a cold fluid is smaller than for the heat capacity rate of a hot fluid. Equations (16) and (17) using to calculate the hot and cold water outlet temperatures at the counter flow double pipe heat exchanger, while equations (18) and (19) using to calculate the hot and cold water outlet temperatures at the parallel flow double pipe heat exchanger. To calculate the heat transfer rate following relations were used:

$$q_h = m_h c_{p,h}(T_{h,i} - T_{h,o}) \qquad (20)$$

$$q_c = m_c c_{p,c}(T_{c,o} - T_{c,i}) \qquad (21)$$

Where c_p specific heat at constant pressure. The heat exchanger efficiency calculates from the following relation:

$$\eta = \frac{q_c}{q_h} \qquad (22)$$

3. The Experimental Setup

The experimental test rig used to determine the double pipe heat exchanger efficiency presented schematically in figure (1). The temperatures of hot and cold water entering and leaving the heat exchanger measured by thermocouples of type T with an accuracy of

±0.3°C. The range of measurement of each flow meter from 0 to 300Lt/hr, with accuracy of ± 2Lt/hr. The inner tube of the heat exchanger is made of copper with inner and outer diameter of 0.0126m and 0.0142m respectively, the outer one is made of galvanized cast iron with inner and outer diameter of 0.0284m and 0.034m respectively with a length of 2.2m. A series of valves 1, 2, 3 and 4 use to select parallel or counter flow heat exchanger.

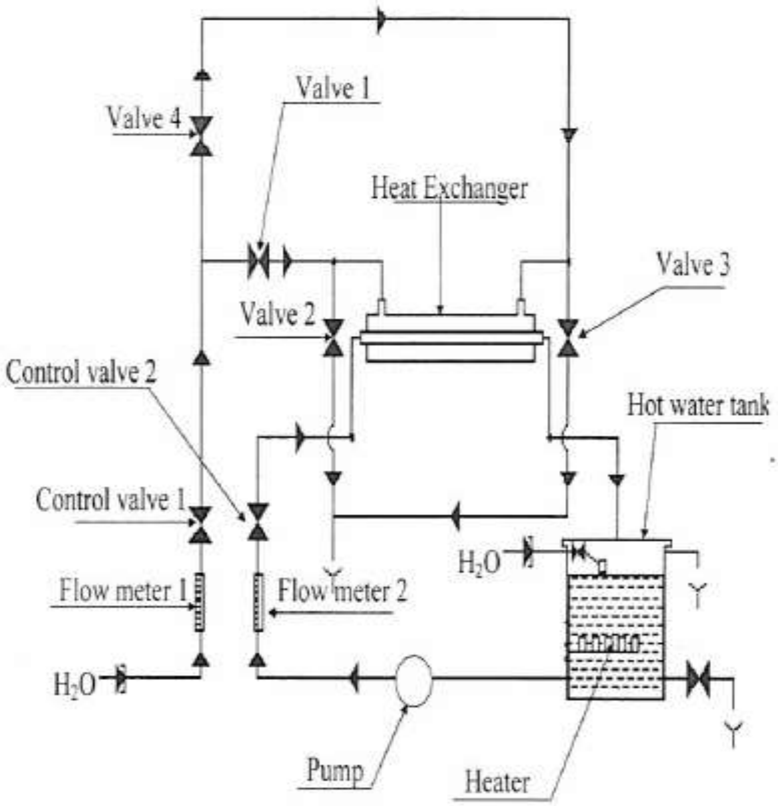
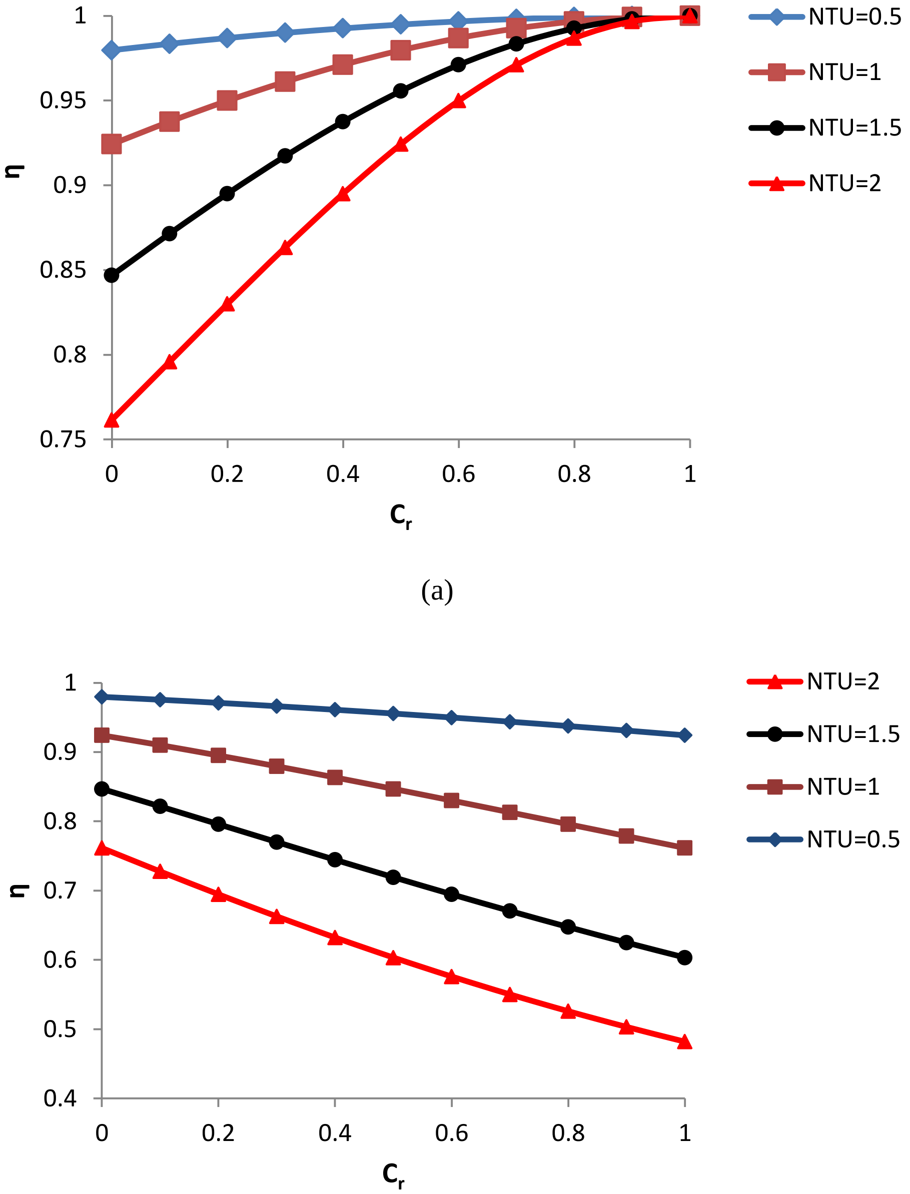


Figure 1. General layout of the experimental test rig.

4. Results and Discussion

The mathematical relationships that derived were used to study the effect of heat capacity ratio and number of transfer units on the efficiency of counter flow and parallel flow heat exchangers. The effect of number of transfer units on efficiency at different values of heat capacity ratio is plotted in figure 2a for counter flow arrangement. It can be noted from the figure that the efficiency decreases as the number of transfer units increases. The effect of number of transfer units on efficiency at different values of heat capacity ratio is plotted in figure 2b for parallel flow arrangement. It can be noted from the figure that the efficiency decreases as the number of transfer units increases. The decreasing in efficiency is higher at high value of NTU.

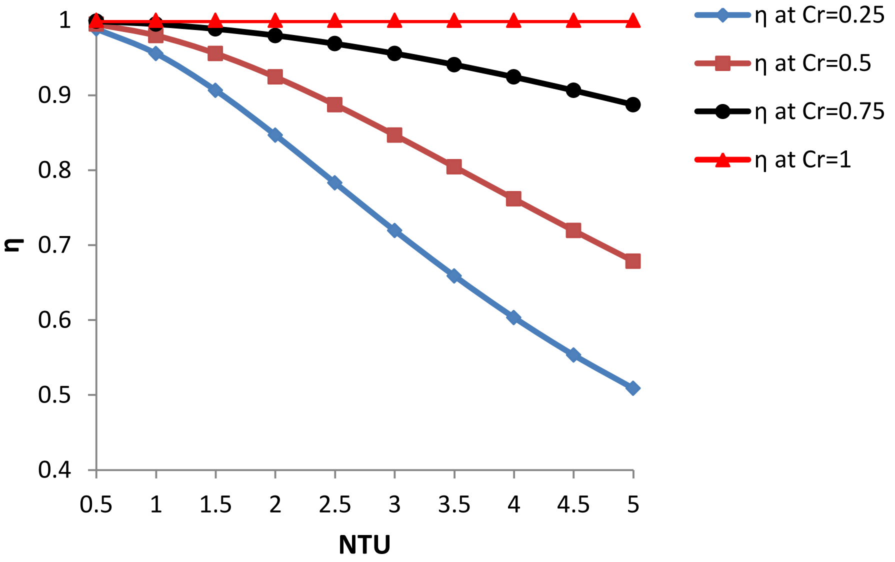


(a)

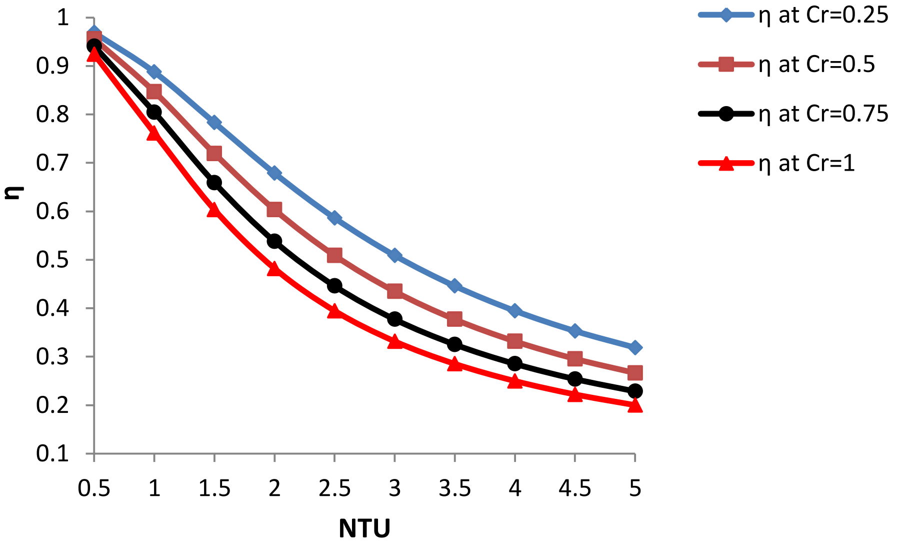
(b)

Figure 2. Effect of NTU on efficiency. (a) Counter flow arrangement. (b) Parallel flow arrangement.

The variation of efficiency with heat capacity ratio for different values of NTU is plotted in figure 3a for counter flow arrangement. It can be noted from the figure that the efficiency increases as the heat capacity ratio increases, and decreasing with the increasing in NTU. The variation of efficiency with heat capacity ratio for different values of NTU is plotted in figure 3b for parallel flow arrangement. It can be noted from the figure that the efficiency decreases as the heat capacity rate increases, and decreasing with the increasing in NTU.



(a)



(b)

Figure 3. Variation of efficiency with C_r. (a) Counter flow arrangement. (b) Parallel flow arrangement.

Figure 4 shows a comparison between the efficiency of the parallel and the counter flow heat exchangers. The efficiency of the counter flow heat exchanger is greater than that for the parallel flow heat exchanger at different values of heat capacity ratio and number of transfer units. The efficiency is higher for lower values of NTU for both parallel flow and counter flow arrangements.

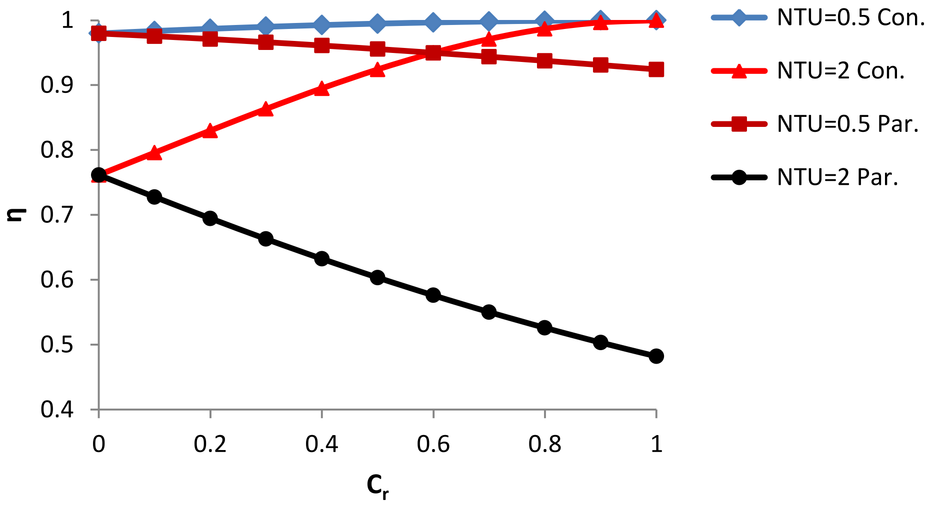


Figure 4. Comparison of counter flow and parallel flow efficiency for different values of NTU.

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The effect of NTU on both efficiency and effectiveness is shown in figure 5 for counter flow arrangement. It can be seen in figure 5 that the curves are away from each other at NTU=0.5, while intersect at NTU=2 and 3, the efficiency at the intersection point when NTU= 3 is greater than that at NTU=2. At the intersection point between efficiency and effectiveness curves, the heat exchanger operates at high value of both efficiency and effectiveness.

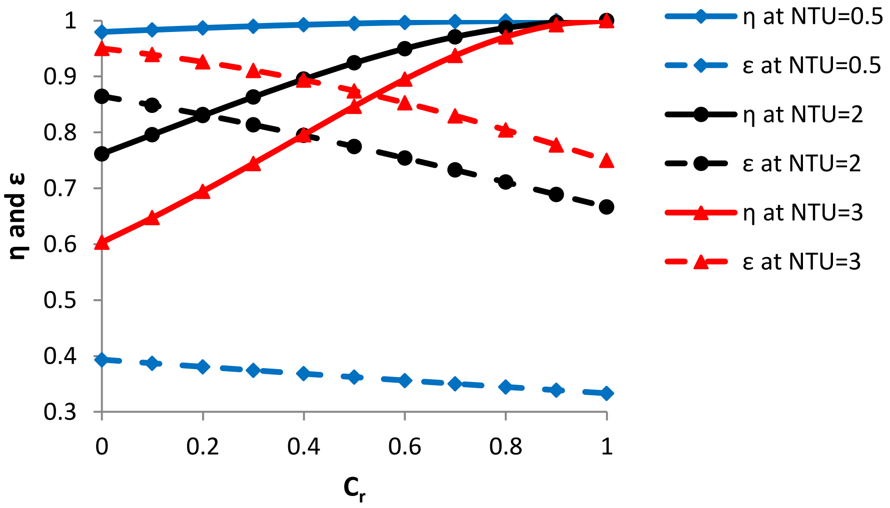

Figure 5. Effect of NTU on efficiency and effectiveness for counter flow arrangement.

The effect of NTU on both efficiency and effectiveness is shown in figure 6 for a parallel flow arrangement. It can be seen in figure 6 that the curves are away from each other, while closer to each other when NTU= 2.

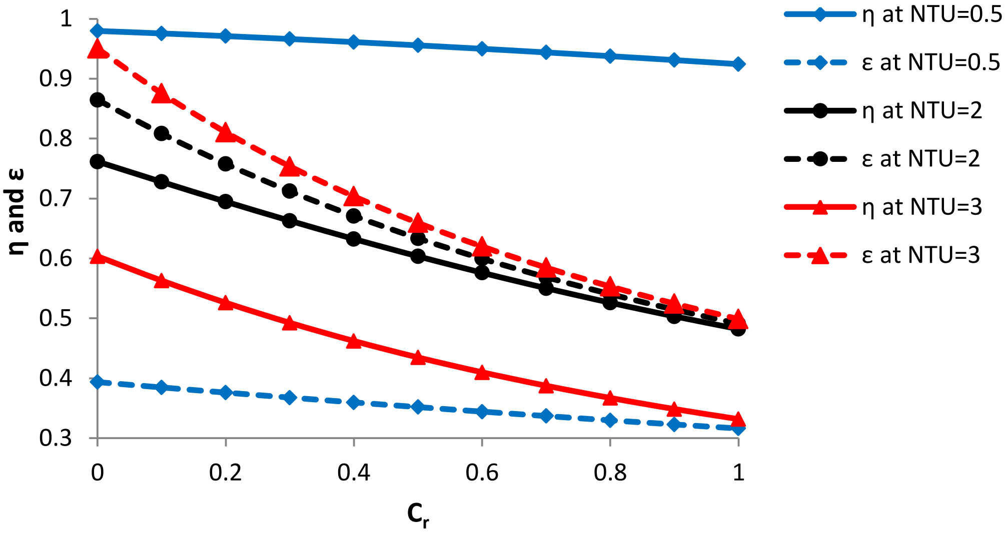

Figure 6. Effect of NTU on efficiency and effectiveness for parallel flow arrangement.

The Variation of both efficiency and effectiveness with C_r is shown in figure 7 for counter flow arrangement. It can be seen in figure 7 that the curves are away from each other at C_r=1, while intersect at C_r=0.25 and 0.5, the efficiency at the intersection point when C_r= 0.5 is greater than that at C_r=0.25.

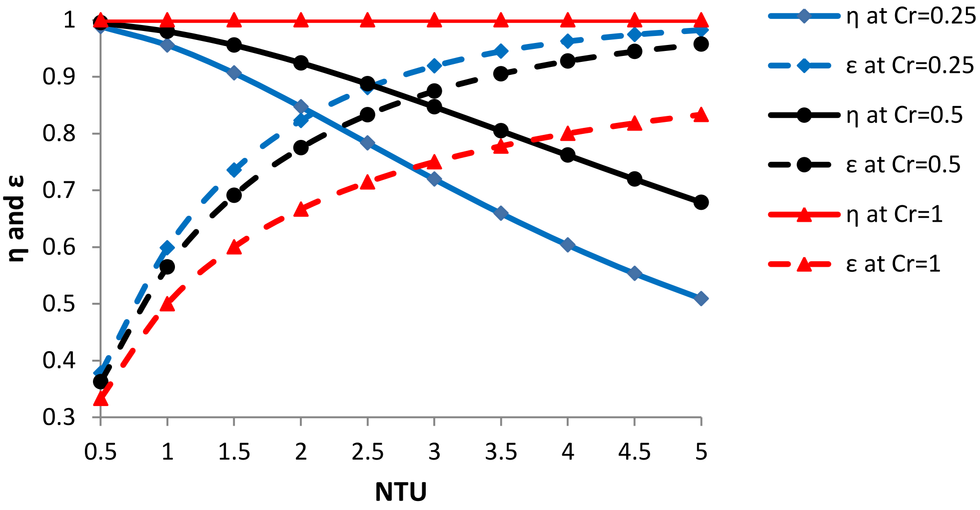

Figure 7. Variation of efficiency and effectiveness with C_r for counter flow arrangement.

The Variation of both efficiency and effectiveness with C_r is shown in figure 8 for parallel flow arrangement. It can be seen in figure 8 that the curves are intersect near NTU=2 at C_r=0.25, 0.5 and 1. The efficiency at the intersection point when C_r= 0.25 is greater than that at C_r=0.5 and 1.

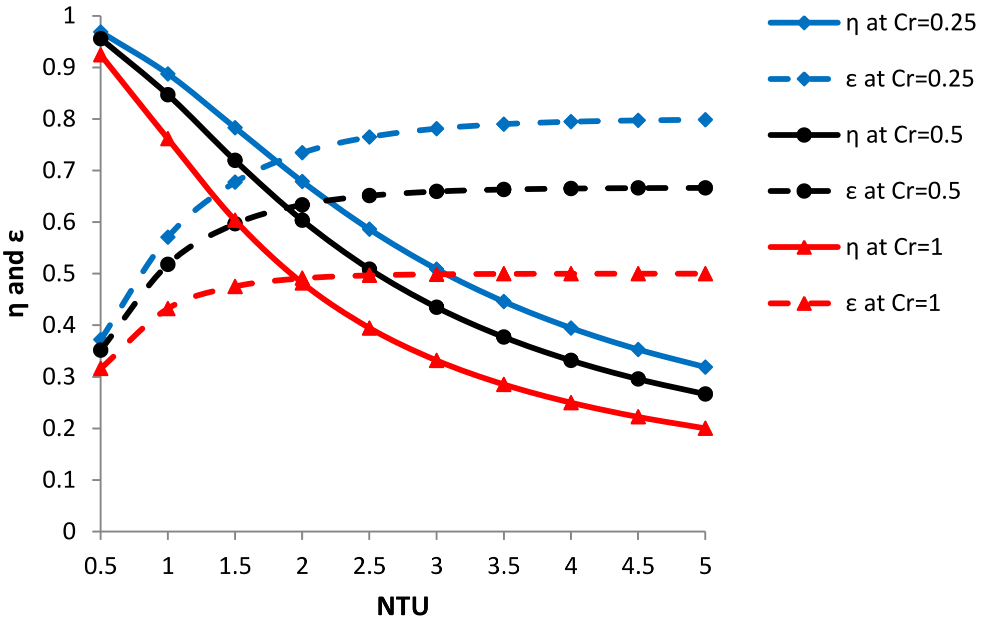

Figure 8. Variation of efficiency and effectiveness with C_r for parallel flow arrangement.

The comparison between analytical and experimental results of the efficiency for the counter flow double pipe heat exchanger is shown in figure 9 at Q_h=250lt/hr, Q_c=100lt/hr, $T_{c,i}$ =23°C, and $T_{h,i}$= 42, 47, 52°C. And in figure 10 for the parallel flow double pipe heat exchanger at Q_h=150, 250, 300lt/hr, Q_c=100lt/hr, $T_{c,i}$ =23°C, and $T_{h,i}$= 42°C. Figures show an agreement between analytical and experimental results.

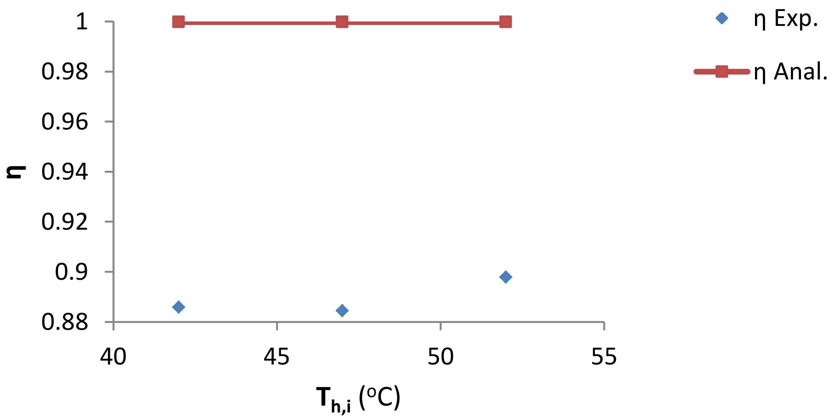

Figure 9. Experimental and analytical efficiency for counter flow heat exchanger.

The equations 4, 5 and 9 were applied to the heat exchanger described as sample problem 10.10 in [5]. The heat exchanger efficiency is found 0.94, the difference between average

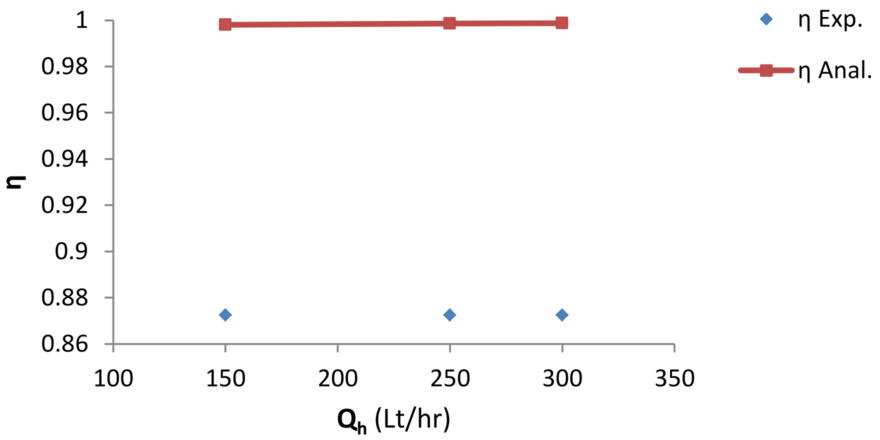

Figure 10. Experimental and analytical efficiency for parallel flow heat exchanger.

temperature of hot and cold fluid is found 32.71°C and the heat transfer rate is found 155.66kW, which is the same answer given in the sample problem 10.10. The equations 4, 5 and 8 were applied to the heat exchanger described as sample problem 10.45 in [6]. The heat exchanger efficiency is found 0.955, the difference between average temperature of hot and cold fluid is found 73.54°C and the heat transfer rate is found 737.61kW, which is the same value can be calculated in the sample problem 10.45.

5. Conclusions

In this paper, new mathematical relationships are presented to determine the efficiency for both parallel and counter flow double pipe heat exchangers. It is based on the value of the heat capacity ratio, and the number of transfer units. The counter flow double pipe heat exchanger efficiency increases by increasing of the heat capacity rate or by decreasing of the number of transfer units. The parallel flow double pipe heat exchanger efficiency increases by decreasing of both heat capacity ratio and number of transfer units. The counter flow double pipe heat exchanger efficiency is greater than that for the parallel flow heat exchanger at different values of heat capacity ratio and number of transfer units.

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